QUESTION 23: EXPLICIT FDM METHOD, IMPLICIT FDM METHOD.

There are two fundamental solutions approaches to solve a parabolic equation: explicit and implicit schemes. Let present this schemes by solving the one dimensional heat equation.

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \tag{1}$$

EXPLICIT METHOD: In this case we need an approximation for the second derivative in space and for the first derivative in time. By using a centered finite difference for the second derivative we have

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{\Delta x^2} \tag{2}$$

For the firs derivative, we use a forward finite difference

$$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^l}{\Delta t} \tag{3}$$

By replacing (2) and (3) into (1) we get

$$k\frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{\Delta x^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$$

Solving for T_i^{l+1} we obtain

$$T_i^{l+1} = T_i^l + k\frac{T_{i+1}^l - 2T_i^l + T_{i-1}^l}{\Delta x^2}\Delta t$$

Let be $\lambda = k\frac{\Delta t}{\Delta x^2}$ then

$$T_i^{l+1} = T_i^l + \lambda\left(T_{i+1}^l - 2T_i^l + T_{i-1}^l\right) \tag{4}$$

Equation (4) can be written for all interior nodes. It provides an explicit solution because for each node we compute values for a future time based on the present values at the node and its neighbors.

Explicit formulations have problem related to stability. Implicit metodhs overcome this difficulty.

IMPLICIT METHOD: In this case the derivative in space is approximated in an advanced time level, then we have

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{\Delta x^2} \tag{5}$$

Notice that in expression (2) we are in time l, in (5) we are in $l+1$. By replacing into equation (1) we get

$$k\frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{\Delta x^2} = \frac{T_i^{l+1} - T_i^l}{\Delta t}$$

therefore

$$T_i^l = T_i^{l+1} - k\frac{T_{i+1}^{l+1} - 2T_i^{l+1} + T_{i-1}^{l+1}}{\Delta x^2}\Delta t$$

Hence

$$T_i^l = -\lambda T_{i+1}^{l+1} + (1+2\lambda)\,T_i^{l+1} - \lambda T_{i-1}^{l+1} \tag{6}$$

for $\lambda = k\frac{\Delta t}{\Delta x^2}$.

Equation (6) is applied for all the internal nodes but for the first and last internal nodes we use the boundary conditions. Hence we obtain the values in the future time by solving the system of equations (6). Since the system is tridiagonal, efficient methods can be used to solve it.