Question 14: Newton's method.

This is perhaps the best known method for finding the zeros of a real function. Let be $f : \mathbb{R} \to \mathbb{R}$,[1] in order to find x^*such that $f(x^*) = 0$, let start with a initial guess x_0 as in figure 1.

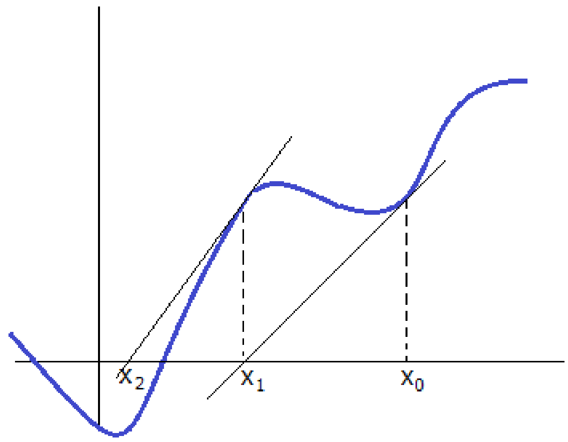

Figure 1: Geometrical interpretation of Newton method.

The Newton method can be derived from a geometrical point of view. Since the first derivative at x is equivalent to the slope

$$f'(x_0) = \frac{f(x_0) - 0}{x_0 - x_1} = \frac{f(x_0)}{x_0 - x_1}$$

Hence $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$, let be $\Delta x = -\frac{f(x_0)}{f'(x_0)}$ then $x_1 = x_0 + \Delta x$. By using $x_1$we find x_2, etc. In general we have

$$x_{i+1} = x_i + \Delta x.$$

The Newthon algorithm is given by:
Step 1: Given a close initial point x_0.
Step 2: For $i = 1, 2, ...$ until convergence.
Step 3: Newton Step $\Delta x = -\frac{f(x_i)}{f'(x_i)}$.
Step 4: Update $x_{i+1} = x_i + \Delta x$.

Advantages of newton method:

- Very simple.
- Fast rate of convergence.

Disadvantages of newton method:

- Is a local method.
- Need first order information.
- We need to solve several linear system.

[1]It can be generalized for $f : \mathbb{R}^n \to \mathbb{R}^n$